



Prefix, Postfix, Infix Notation

Infix Notation

- To add A, B, we write
 $A+B$
- To multiply A, B, we write
 $A*B$
- The operators ('+' and '*') go in between the operands ('A' and 'B')
- This is "*Infix*" notation.

Prefix Notation

- Instead of saying "A plus B", we could say "add A,B" and write
 $+ A B$
- "Multiply A,B" would be written
 $* A B$
- This is *Prefix* notation.

Postfix Notation

- Another alternative is to put the operators after the operands as in
 $A B +$
- and
 $A B *$
- This is *Postfix* notation.

Pre A In B Post

- The terms infix, prefix, and postfix tell us whether the operators go between, before, or after the operands.

Parentheses

- Evaluate $2+3*5$.
- + First:
 $(2+3)*5 = 5*5 = 25$
- * First:
 $2+(3*5) = 2+15 = 17$
- Infix notation requires Parentheses.

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What about Prefix Notation?

- $+ 2 * 3 5 =$
 $= + 2 * 3 5$
 $= + 2 15 = 17$
- $* + 2 3 5 =$
 $= * + 2 3 5$
 $= * 5 5 = 25$
- No parentheses needed!

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Postfix Notation

- $2 3 5 * + =$
 $= 2 3 5 * +$
 $= 2 15 + = 17$
- $2 3 + 5 * =$
 $= 2 3 + 5 *$
 $= 5 5 * = 25$
- No parentheses needed here either!

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Conclusion:

- Infix is the only notation that requires parentheses in order to change the order in which the operations are done.

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Fully Parenthesized Expression

- A FPE has exactly one set of Parentheses enclosing each operator and its operands.
- Which is fully parenthesized?
 $(A + B) * C$
 $((A + B) * C)$
 $((A + B) * (C))$
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Infix to Prefix Conversion

Move each operator to the left of its operands & remove the parentheses:

$$((A + B) * (C + D))$$

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Infix to Prefix Conversion

Move each operator to the left of its operands & remove the parentheses:

$$(+ A B * (C + D))$$

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Infix to Prefix Conversion

Move each operator to the left of its operands & remove the parentheses:

$$* + A B (C + D)$$

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Infix to Prefix Conversion

Move each operator to the left of its operands & remove the parentheses:

$$* + A B + C D$$

Order of operands does not change!

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Infix to Postfix

$$(((A+B)*C)-((D+E)/F))$$

$$A B + C * D E + F / -$$

- Operand order does not change!
- Operators are in order of evaluation!

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Summary of the rules follows:

1. Print operands as they arrive.
2. If the stack is empty or contains a left parenthesis on top, push the incoming operator onto the stack.
3. If the incoming symbol is a left parenthesis, push it on the stack.
4. If the incoming symbol is a right parenthesis, pop the stack and print the operators until you see a left parenthesis. Discard the pair of parentheses.

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5. If the incoming symbol has higher precedence than the top of the stack, push it on the stack.
6. If the incoming symbol has equal precedence with the top of the stack, use association. If the association is left to right, pop and print the top of the stack and then push the incoming operator. If the association is right to left, push the incoming operator.
7. If the incoming symbol has lower precedence than the symbol on the top of the stack, pop the stack and print the top operator. Then test the incoming operator against the new top of stack.
8. At the end of the expression, pop and print all operators on the stack. (No parentheses should remain.)

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Infix to postfix conversion

INFIX

$$(a + b - c) * d - (e + f)$$

POST

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Stack

INFIX
 $a + b - c) * d - (e + f)$

POSTFIX

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Stack

INFIX
 $+ b - c) * d - (e + f)$

POSTFIX
 a

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Stack Infix to postfix conversion

Stack

INFIX
 $b - c) * d - (e + f)$

POSTFIX
 a

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Stack

INFIX
 $- c) * d - (e + f)$

POSTFIX
 $a b$

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Stack

INFIX
 $c) * d - (e + f)$

POSTFIX
 $a b +$

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Stack

INFIX
 $) * d - (e + f)$

POSTFIX
 $a b + c$

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Stack

INFIX
* d - (e + f)

POSTFIX
a b + c -

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Stack

INFIX
d - (e + f)

POSTFIX
a b + c -

*

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Stack

INFIX
- (e + f)

POSTFIX
a b + c - d

*

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Stack

INFIX
(e + f)

POSTFIX
a b + c - d *

-

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Stack

INFIX
e + f)

POSTFIX
a b + c - d *

(

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Stack

INFIX
+ f)

POSTFIX
a b + c - d * e

(

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Infix to postfix conversion

Stack

INFIX
f)

POSTFIX
a b + c - d * e

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Infix to postfix conversion

Stack

INFIX
)

POSTFIX
a b + c - d * e f

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Infix to postfix conversion

Stack

INFIX
[]

POSTFIX
a b + c - d * e f +

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Infix to postfix conversion

stack

INFIX
[]

POSTFIX
a b + c - d * e f + -

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INFIX	POSTFIX
$(A + B) * C + D / (E + F * G) - H$	
$(A * B + C) / D - E / (F + G)$	
$A - B - C * (D + E / F - G) - H$	
$A + ((B - C * D) / E) + F - G / H$	
$(A * B - (C - D)) / (E + F)$	
$A * B ^ A C + D$	

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INFIX	POSTFIX
$(A + B) * C + D / (E + F * G) - H$	$AB + C * DEFG * + / + H -$
$(A * B + C) / D - E / (F + G)$	$AB * C + D / EFG + / -$
$A - B - C * (D + E / F - G) - H$	$AB - CDEF / + G * - H -$
$A + ((B - C * D) / E) + F - G / H$	$ABCD * - E / + F + GH / -$
$(A * B - (C - D)) / (E + F)$	$AB * CD - EF + /$
$A * B ^ A C + D$	$ABC ^ A * D +$

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$(A + B) * C + D / (E + F * G) - H$

INPUT	STACK	POSTFIX
((
(((
A	((A
+	((+	A
B	((+	AB
)	(AB+
*	(*	AB+
C	(*	AB+C
+	(+)	AB+C+
D	(+)	AB+C*D
/	(+)	AB+C*D
((+/(AB+C*D

VKS-LEARNING_HUB C

INPUT	STACK	POSTFIX
E	(+/(AB+C*DE
+	(+/(+	AB+C*DE
F	(+/(+	AB+C*DEF
*	(+/(+*	AB+C*DEF
G	(+/(+*	AB+C*DEFG
)	(+/(AB+C*DEFG+
-	(-	AB+C*DEFG*+/-
H	(-	AB+C*DEFG*+/-H
)	EMPTY	AB+C*DEFG*+/-H-

$AB+C*DEFG*+/-H-$

VKS-LEARNING_HUB C

$(A * B + C) / D - E / (F + G)$

INPUT	STACK	POSTFIX
(
(
A		
*		
B		
+		
C		
)		
/		
D		
-		
E		

VKS-LEARNING_HUB C

$(A * B + C) / D - E / (F + G)$

INPUT	STACK	POSTFIX
/		
(
F		
+		
G		
)		
)		

$AB*C+D/EF+/-$

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Evaluation of Postfix notation

The Postfix notation is used to represent algebraic expressions. The expressions written in postfix form are evaluated faster compared to infix notation as parenthesis are not required in postfix.

Keep the following points for evaluation postfix expressions.

- 1) Create a stack to store operands (or values).
- 2) Scan the given expression and do following for every scanned element.
 - If the element is a **number**, **push** it into the stack
 - If the element is a **operator**, **pop** operands for the operator from stack. Evaluate the operator and **push the result back to the stack**
- 3) When the expression is ended, the number in the stack is the final answer

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Infix Expression **$(5 + 3) * (8 - 2)$**

Postfix Expression **$5 3 + 8 2 - *$**

Below Postfix Expression can be evaluated by using Stack Data Structure as follows.

Reading Symbol	Stack Operations	Evaluated Part of Expression
Initially	Stack is Empty	Nothing
5	push(5)	Nothing

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3	push(3)		Nothing
+	value1 = pop() value2 = pop() result = value2 + value1 push(result)		value1 = pop(); // 3 value2 = pop(); // 5 result = 5 + 3; // 8 Push(8) (5 + 3)
8	push(8)		(5 + 3)

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2	push(2)		(5 + 3)
-	value1 = pop() value2 = pop() result = value2 - value1 push(result)		value1 = pop(); // 2 value2 = pop(); // 8 result = 8 - 2; // 6 Push(6) (8 - 2) (5 + 3), (8 - 2)
*	value1 = pop() value2 = pop() result = value2 * value1 push(result)		value1 = pop(); // 6 value2 = pop(); // 8 result = 8 * 6; // 48 Push(48) (6 * 8) (5 + 3) * (8 - 2)

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\$	End of Expression	result = pop()		Display (result) 48 As final result
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Infix Expression $(5 + 3) * (8 - 2) = 48$
 Postfix Expression $5\ 3\ +\ 8\ 2\ -\ *\$ value is **48**

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4 5 + 9 * 3 + 3 /		
INPUT	ACTION	STACK
4	PUSH 4 TO STACK (operand are pushed to stack)	4
5	PUSH 5 TO STACK(operand are pushed to stack)	5
+	POP 5 AND POP 4 AND CALCULATE VALUE WITH OPERATOR (4+5) AND PUSH THE RESULT TO STACK	9
9	PUSH 9 TO STACK (operand are pushed to stack)	9
*	POP 9 AND POP 9 AND CALCULATE VALUE WITH OPERATOR (9*9) AND PUSH THE RESULT TO STACK	81
3	PUSH 3 TO STACK	3
+	POP 3 AND POP 81 AND CALCULATE VALUE WITH OPERATOR (81+3) AND PUSH THE RESULT TO STACK	84
3	PUSH 3 TO STACK	3
/	POP 3 AND POP 84 AND CALCULATE VALUE WITH OPERATOR (84/3) AND PUSH THE RESULT TO STACK	28

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54 6 + 7 4 - * 9 / 35 15 + +		
INPUT	ACTION	STACK
54		
6		
+		
7		
4		
-		
*		
9		
/		
35		
+		
+		
		70